Intermittency and energy cascade in helical-decimated Navier-Stoke's equations

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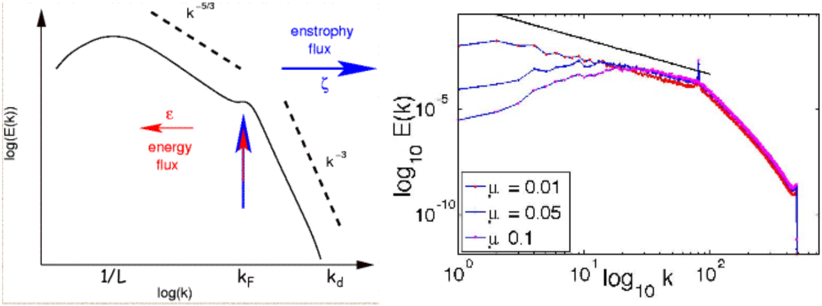
Outline

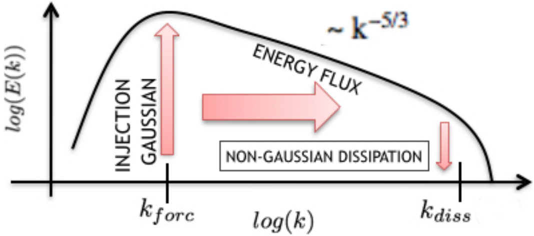
- Introduction
- Numerics
- Results
- Conclusions

- For 2D Navier-Stokes equations two conserved quantities: Energy $E = \int d^2r\ \vec{u} \cdot \vec{u}$ and Enstrophy $\Omega = \int d^2r\ \vec{\omega} \cdot \vec{\omega}$
- Forward cascade of energy is blocked, since enstrophy is also positive and definite. (Boffetta Ann. Rev. Fluid Mech 2012)



Ray et al, Phys. Rev. Lett. 107, 184503 (2011)

- The invariants of 3D Navier-Stokes equations:
  Energy $E = \int d^3r\ \vec{u} \cdot \vec{u}$ and Helicity $H = \int d^3r\ \vec{u} \cdot \vec{\omega}$
- Helicity could be positive or negative.
- Both cascades forward, from large scales to small scales. (Chen, Phys. Fluids 2003)



- Growth of helicity at small scales, both in positive and negative modes but finite because of the mirror symmetry.

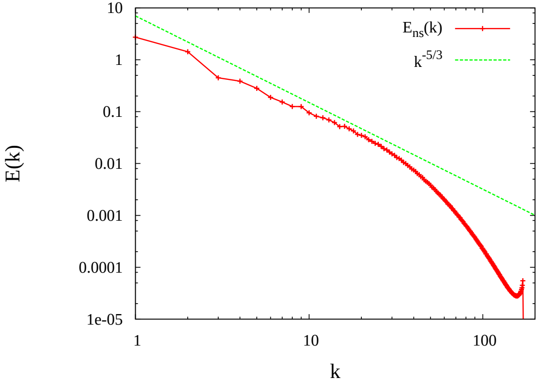
3D Navier-Stokes equation for incompressible flows

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = \nu \Delta \mathbf{u} - \nabla p + \mathbf{f};\ \ \nabla \cdot \mathbf{u} = 0.$$

- Homogeneous and isotropic turbulence.
- Numerically solved in a 3D periodic domain using Gaussian delta-correlated forcing.
- Forward energy cascade from large scales to small scales.
- Non-Gaussian PDFs show longer tails as indication of intermittency.

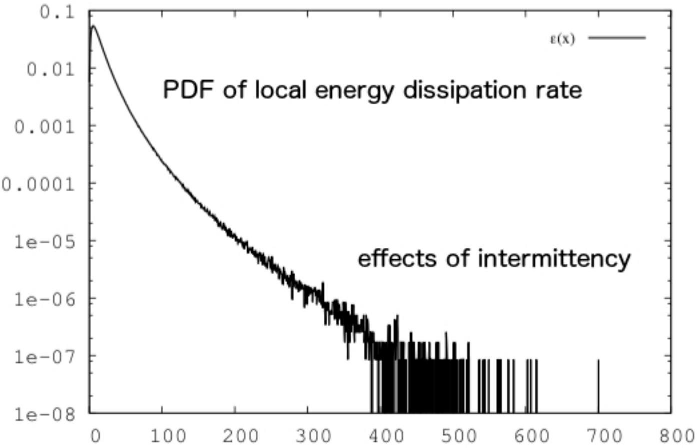
Energy spectrum

Energy spectra $E(k) = \sum_{\mathbf{k} \ni |\mathbf{k}|=k} |\mathbf{u}(\mathbf{k})|^2$



- Forward energy cascade from large scales to small scales in our DNS of 3D Navier-Stokes equations.
- Shows a Kolmogorov $k^{-5/3}$ scaling in the inertial range.

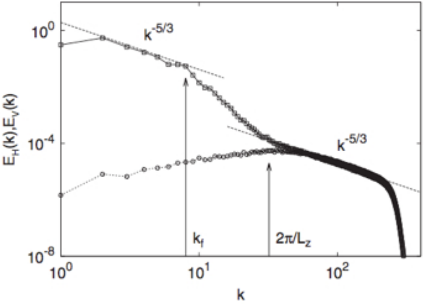
PDF of local energy dissipation rate



- Non-Gaussian PDFs show longer tails as indication of intermittency.

Evidence of inverse energy cascade

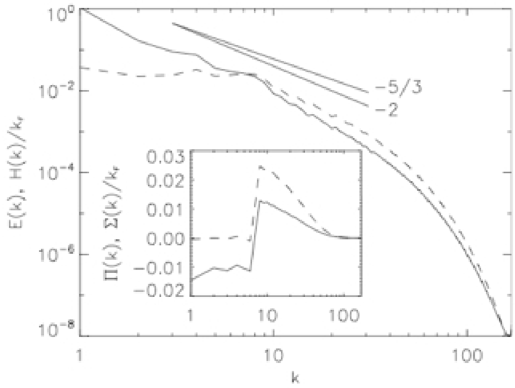
- Energy spectrum in a turbulent flow confined in thick fluid layers. (Xia et al, Nat Phys 2011)



- Formation of large scale vortex suppresses vertical motion and supports large scale energy transfer.

Evidence of inverse energy cascade

P.D. Mininni and A. Pouquet. Phys. Rev. E 79, 026304 (2009)



- Energy spectrum in rotational turbulence with helical force.
- Plots of fluxes in inset shows direct cascade of helicity and direct and inverse cascade of enegy.
- Positive definiteness of helicity leads to inverse energy tranfer.

Dynamics of inverse cascade of energy is a subset of all interactions in the NS equations.

- What happens when we change the relative weight between positive and negative helicity modes?
- Can we separate positive and negative modes to understand the dynamics?

- In Fourier space, $\mathbf{u}(\mathbf{k}, t)$ has two degrees of freedom since $\mathbf{k} \cdot \mathbf{u}(\mathbf{k}, t) = 0$.
- We chose projection on orthonormal helical waves with definite sign of helicty.
- Following Waleffe Phys. Fluids (1992)

$$\mathbf{u}(\mathbf{k}, t) = a^{+}(\mathbf{k}, t)\mathbf{h}^{+}(\mathbf{k}) + a^{-}(\mathbf{k}, t)\mathbf{h}^{-}(\mathbf{k})$$

  where $\mathbf{h}^{\pm}(\mathbf{k})$ are the complex eigenvectors of the curl operator $i\mathbf{k} \times \mathbf{h}^{\pm}(\mathbf{k}) = \pm k\mathbf{h}^{\pm}(\mathbf{k})$.
- $\mathbf{h}_s^* \cdot \mathbf{h}_t = 2\delta_{st}$; $\mathbf{h}_s^* = \mathbf{h}_{-s}$,
  where s and t could be $+1$ or -1

Helical-decomposition of velocity

- Choose $\mathbf{h}^{\pm}(\mathbf{k}) = \hat{\mu}(\mathbf{k}) \times \hat{\mathbf{k}} \pm i\hat{\mu}$,
  where $\hat{\mu}$ is an arbitrary unit vector orthogonal to $\mathbf{k}$
- reality of the velocity field requires $\hat{\mu}(\mathbf{k}) = -\hat{\mu}(-\mathbf{k})$
- Such requirement is satisfied, e.g., by the choice $\hat{\mu}(\mathbf{k}) = \mathbf{z} \times \mathbf{k}/||\mathbf{z} \times \mathbf{k}||$, with $\mathbf{z}$ an arbitrary vector.
- Projection operator:

$$\mathcal{P}^{\pm}(\mathbf{k}) \equiv \frac{\mathbf{h}^{\pm}(\mathbf{k}) \otimes \mathbf{h}^{\pm}(\mathbf{k})^*}{\mathbf{h}^{\pm}(\mathbf{k})^* \cdot \mathbf{h}^{\pm}(\mathbf{k})}$$

$$\mathbf{u}^{\pm}(\mathbf{k}, t) = \mathcal{P}^{\pm}(\mathbf{k})\mathbf{u}(\mathbf{k}, t)$$

$$\mathbf{u}(\mathbf{k}, t) = \mathbf{u}^{+}(\mathbf{k}, t) + \mathbf{u}^{-}(\mathbf{k}, t)$$

- Eneregy $E(t) = \sum_{\mathbf{k}} |\mathbf{u}^{+}(\mathbf{k}, t)|^2 + |\mathbf{u}^{-}(\mathbf{k}, t)|^2$.
- Helicity $\mathcal{H}(t) = \sum_{\mathbf{k}} k(|\mathbf{u}^{+}(\mathbf{k}, t)|^2 - |\mathbf{u}^{-}(\mathbf{k}, t)|^2)$.

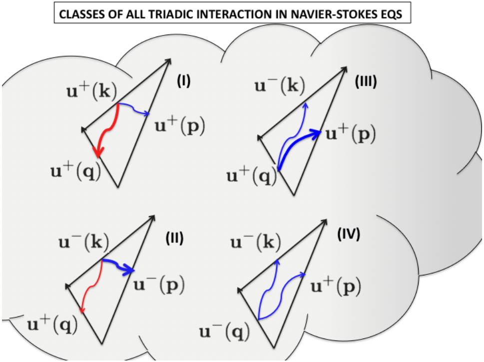
- Decimated Navier-Stokes equations in Fourier space:

$$\partial_t \mathbf{u}^{\pm}(\mathbf{k}, t) = \mathcal{P}^{\pm}(\mathbf{k})\mathbf{N}_{u^{\pm}}(\mathbf{k}, t) + \nu k^2 \mathbf{u}^{\pm}(\mathbf{k}, t) + \mathbf{f}^{\pm}(\mathbf{k}, t)$$

  where ν is kinematic viscosity and $\mathbf{f}$ is external forcing.
- The nonlinear term containing all triadic interactions

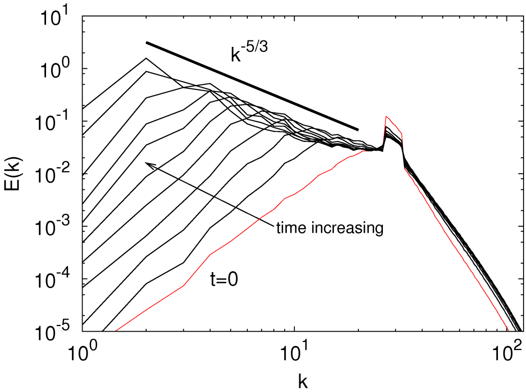
$$\mathbf{N}_{u^{\pm}}(\mathbf{k}, t) = \mathcal{F}T(\mathbf{u}^{\pm} \cdot \nabla \mathbf{u}^{\pm} - \nabla p)$$

Helical-decimated Navier-Stokes equations



- Four classes of nonlinear triadic interactions with definite helicity signs under helical decomposition of NS equations.
- Energy and helicity are conserved for each triads.

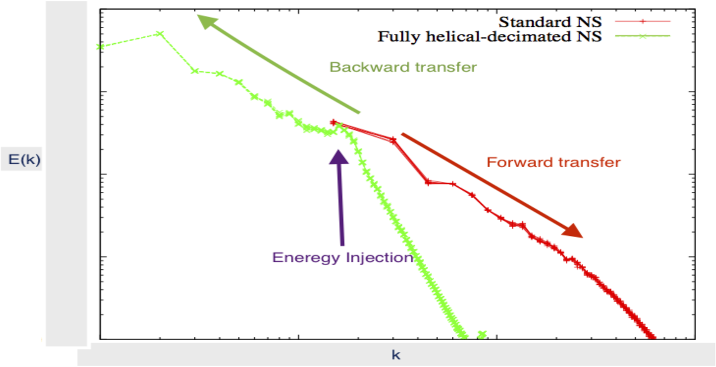
Triads with only $\mathbf{u}^{+}$ lead to reversal of energy cascade.



Energy spectra in the inverse cascade regime shows $k^{-5/3}$ slope.

Biferale PRL (2012)

- Full decimation of $\mathbf{u}^+$ or $\mathbf{u}^-$ $\rightarrow$ inverse cascade of energy.
  No decimation $\rightarrow$ forward cascade of energy



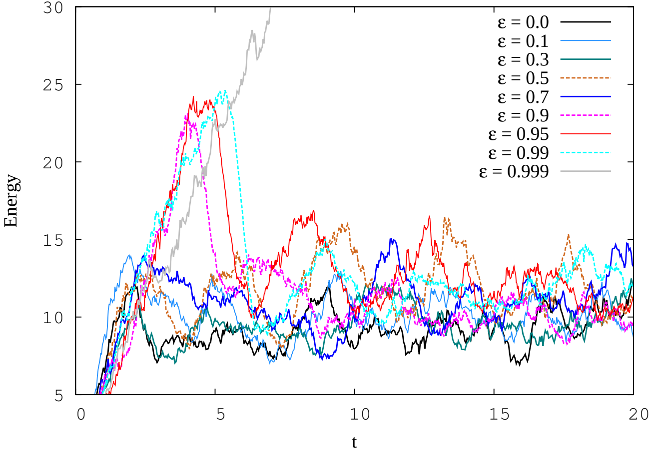
- What happens in between??
- We decided not to kill all modes of a particular sign, but a fraction ϵ of them.
- Is there a *Critical value* of ϵ at which forward cascade of energy stops?
  alternatively, inverse cascade of energy stops if forced at small scales.

- Modified projection operator:

$$\mathcal{P}_\epsilon^+(\mathbf{k})\mathbf{u}(\mathbf{k},t) = \mathbf{u}^+(\mathbf{k},t) + \theta_\epsilon(\mathbf{k})\mathbf{u}^-(\mathbf{k},t)$$

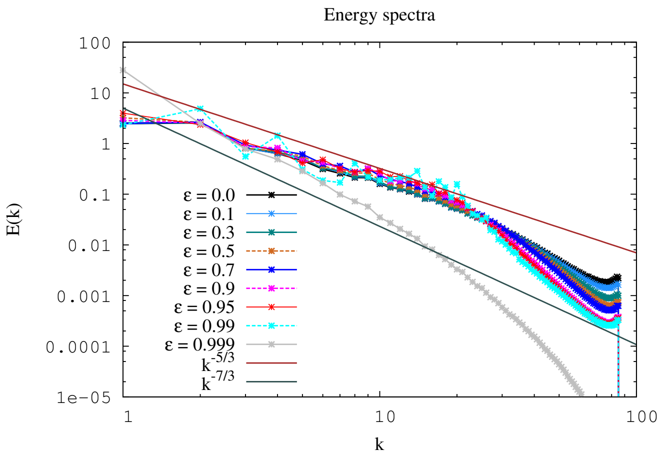
  where $\theta_\epsilon(\mathbf{k})$ is 0 or 1 with probability ϵ and $1-\epsilon$, respectively.
- $\epsilon = 0 \rightarrow$ Standard Navier-Stokes.
  $\epsilon = 1 \rightarrow$ Fully helical-decimated Navier-Stokes.
- Pseudo-spectral DNS on a triply periodic cubic domain of size $L = 2\pi$ with resolutions upto 512^3 collocation points.
- Random Gaussian forcing:
  $\langle f_i(\mathbf{k},t) f_j(\mathbf{q},t')\rangle = F(k)\delta(\mathbf{k}-\mathbf{q})\delta(t-t')Q_{i,j}(\mathbf{k})$,
  where $Q_{ij}(\mathbf{k})$ is a projector assuring incompressibility.
  $F(k)$ is nonzero only in the low wavenumber range
  $|k| \in [1:2]$.

Evolution of energy



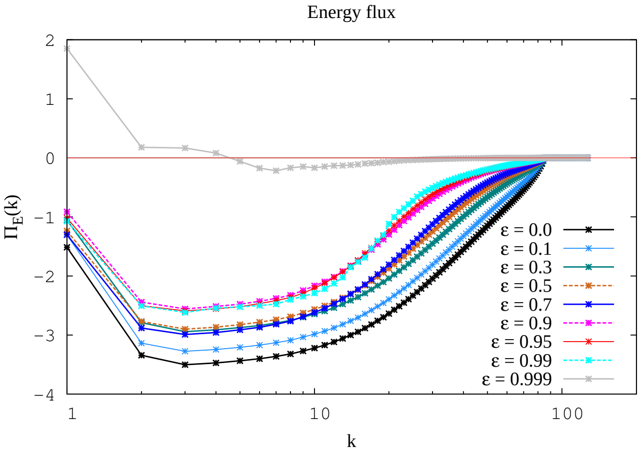
Energy vs time shows a initial block at the large scales before reaching a steady state.
With increase in ϵ the peak grows, a signature of inverse cascade.

Energy spectra



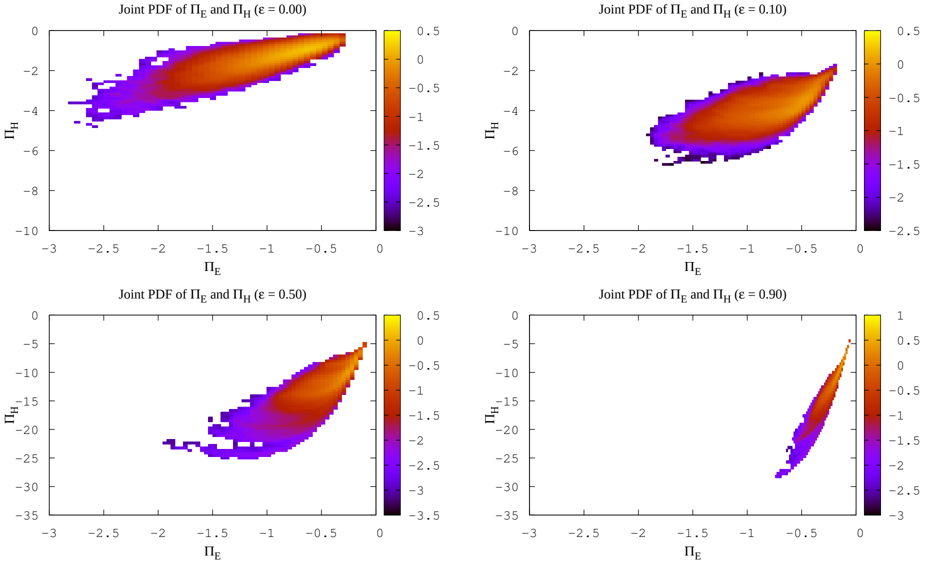
At $\epsilon = 0.99$ the spectrum shows large fluctuations. (Critical Value!)
At $\epsilon = 0.999$ forward cascade of energy subsides.
and Energy spectra shows a $k^{-7/3}$ spectrum due to forward helicity cascade.

Energy flux



Energy flux gets depleted in the small scales with increasing ϵ.
There is sudden reversal of flux as we change ϵ from 0.99 to 0.999.

Joint PDF of helicity and energy fluxes

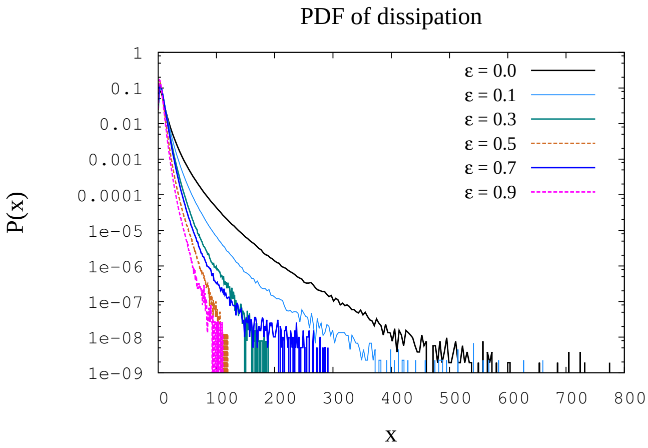


The helicity flux attains higher values whereas the energy flux depletes with increasing ϵ.

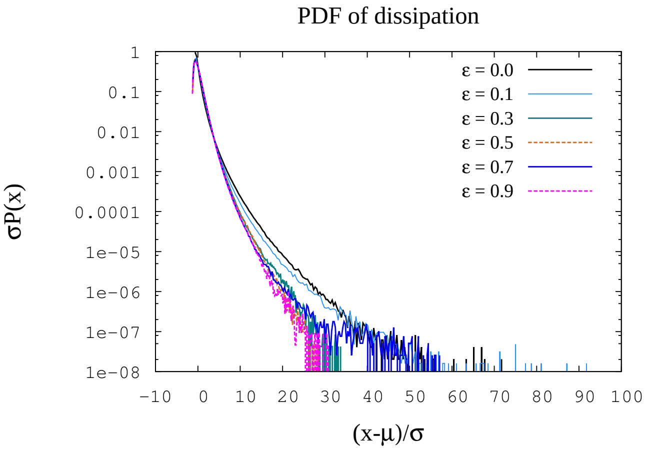
- As we increase ϵ, the contribution of triads leading to inverse energy cascade grows.
- Only when ϵ is very close to 1 inverse energy cascade takes over the forward cascade.
- Critical value of ϵ may have Reynolds number dependence! We are attempting high resolution DNS to cover a range of Reynolds numbers.
- Can both forward and inverse cascade co-exist? We made simulations with forcing in the inertial range.
- What about intermittency in the forward cascade regime at changing ϵ.

Let us look at some more statistics...

local energy dissipation rate



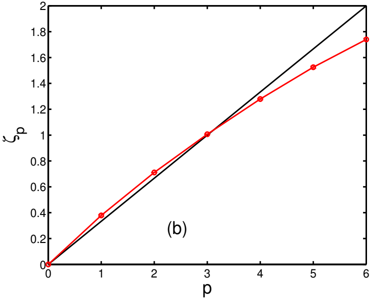
Comparison of PDFs of local energy dissipation rates show reduction of longer tails with increase in fraction of decimation ϵ. Less of extreme dissipation events show decrease in intermittency with increaseing ϵ



Comparison of standardized PDFs of local energy dissipation rates show decrease in intermittency with increasing ϵ

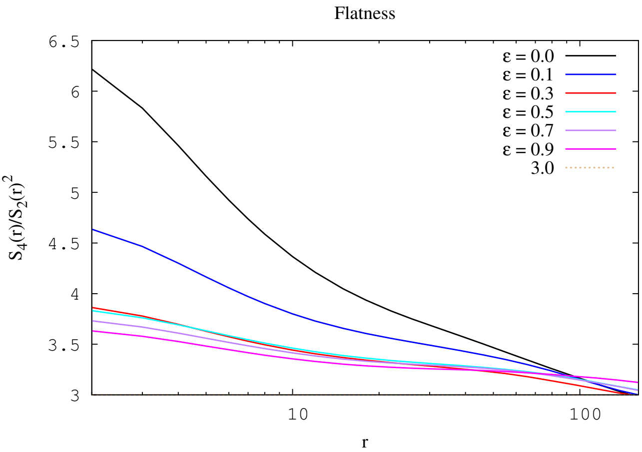
Structure functions

- Order-p equal-time, longitudinal velocity structure functions $S_p(r) \equiv \langle |\delta u_{\|}(\mathbf{x}, r)|^p \rangle$ where $\delta u_{\|}(\mathbf{x}, r) \equiv [\mathbf{u}(\mathbf{x}+\mathbf{r}, t) - \mathbf{u}(\mathbf{x}, t)] \cdot \frac{\mathbf{r}}{r}$
- In the inertial range we see the universal scaling $S_p(r) \sim r^{\zeta_p}$



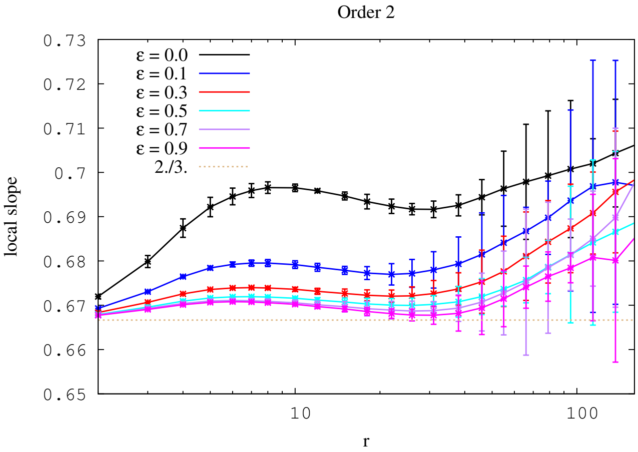
- Deviations from Kolmogorov scaling $\zeta_p^{K41} = p/3$ shows present intermittency.
- Extended Self-Similarity: ζ_p/ζ_3.

Measure of intermittency: Flatness $F_4(r) = S_4(r)/[S_2(r)]^2$



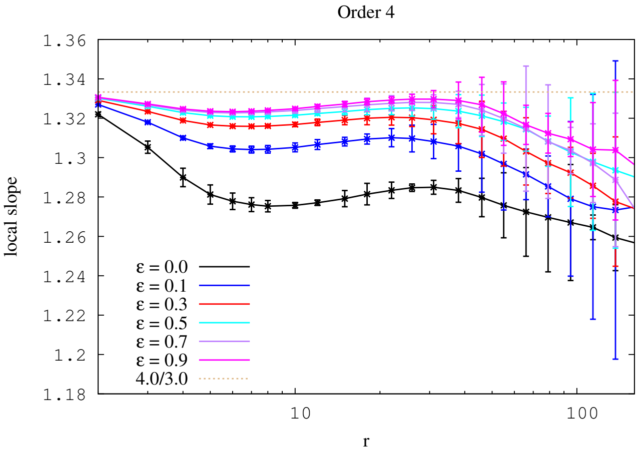
- Measure of flatness shows the small scale intermittency reduces significantly when 10% of $\mathbf{u}^-$ modes are killed.
- It reduces further and seems saturated with increase in ϵ

Measure of intermittency: ζ_2



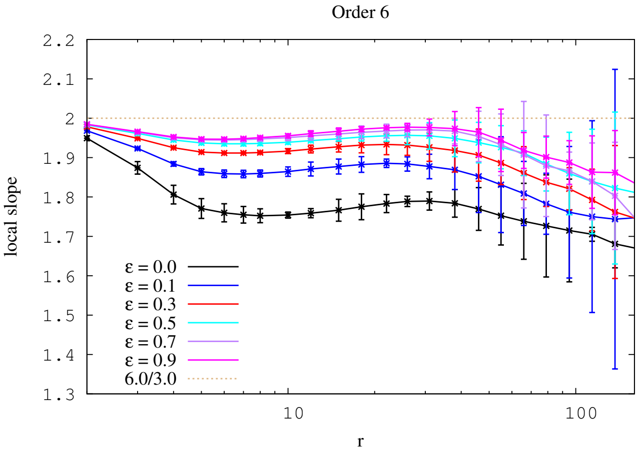
Multiscaling exponent using ζ_2 ESS, local slope of $S_2(r)/S_3(r)$ increases with increase in ϵ

Measure of intermittency: ζ_4



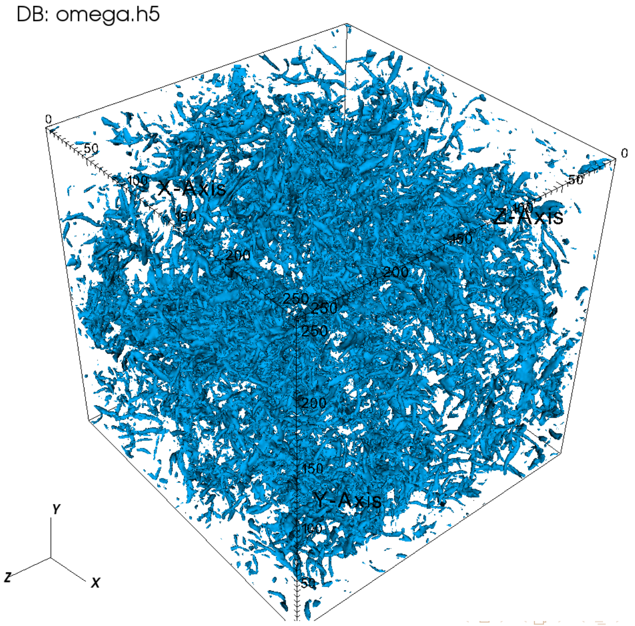
Multiscaling exponent using ζ_4 ESS, local slope of $S_4(r)/S_3(r)$ decreases with increase in ϵ

Measure of intermittency: ζ_6

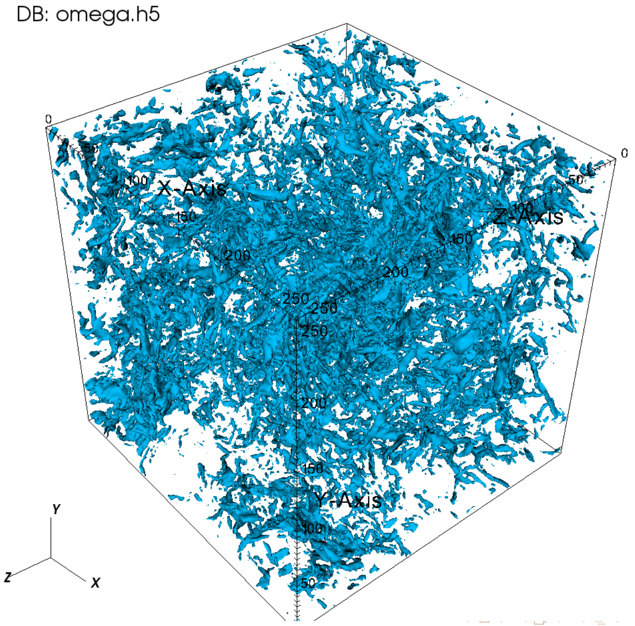


Multiscaling exponent using ζ_6 ESS, local slope of $S_6(r)/S_3(r)$ decreases with increase in ϵ

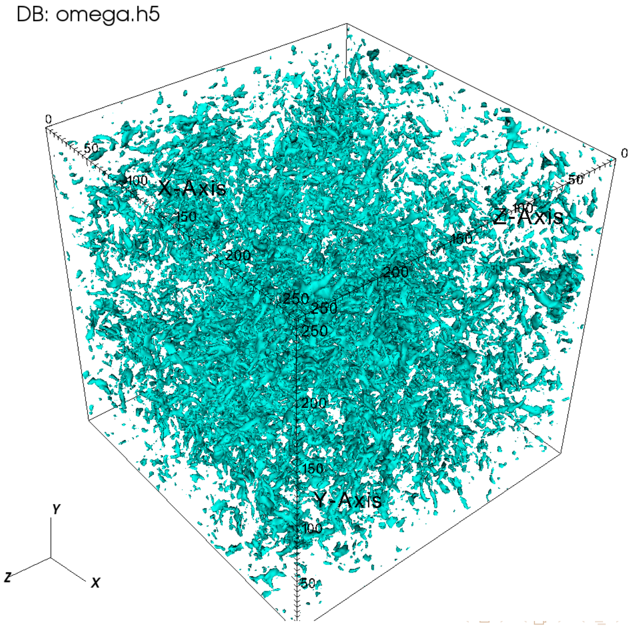
Isovorticity surfaces, $\epsilon = 0$



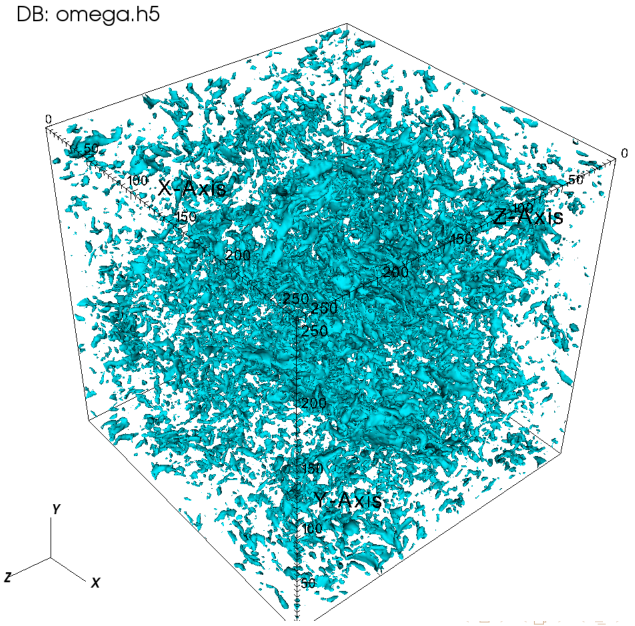
Isovorticity surfaces, $\epsilon = 0.1$



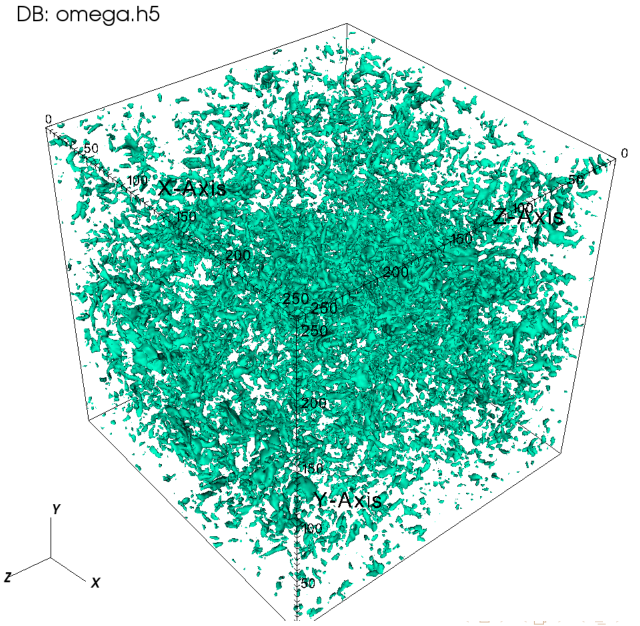
Isovorticity surfaces, $\epsilon = 0.3$



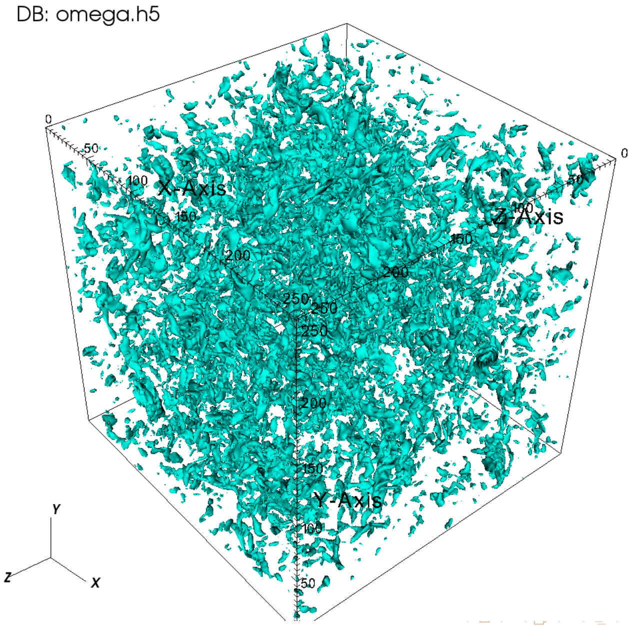
Isovorticity surfaces, $\epsilon = 0.5$



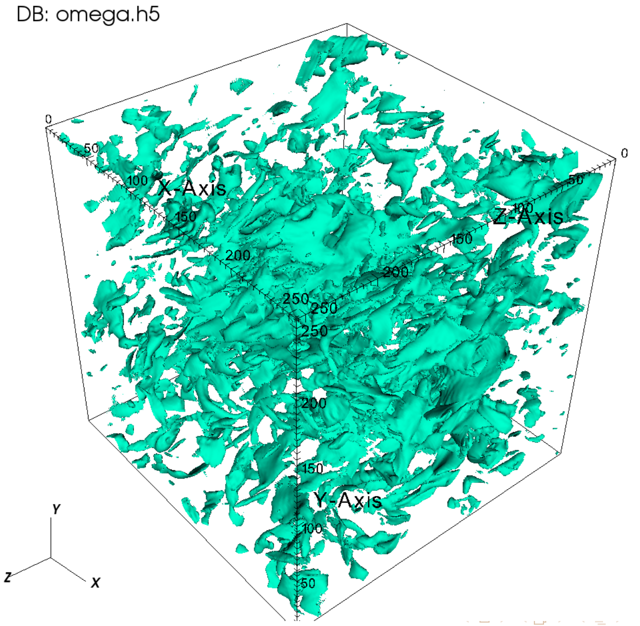
Isovorticity surfaces, $\epsilon = 0.7$



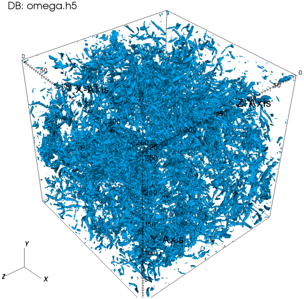
Isovorticity surfaces, $\epsilon = 0.9$

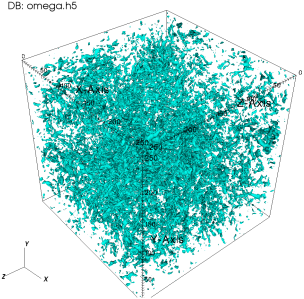


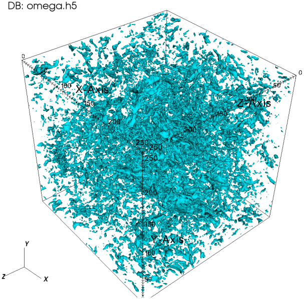
Isovorticity surfaces, $\epsilon = 0.999$

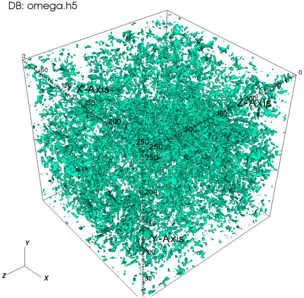


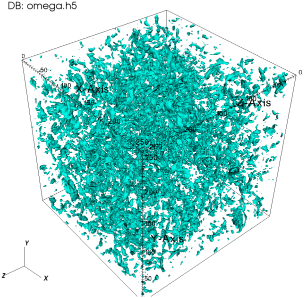
Isovorticity surfaces

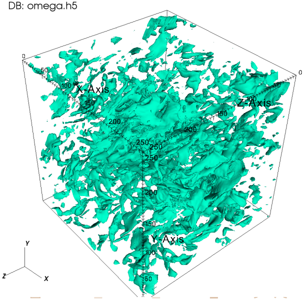








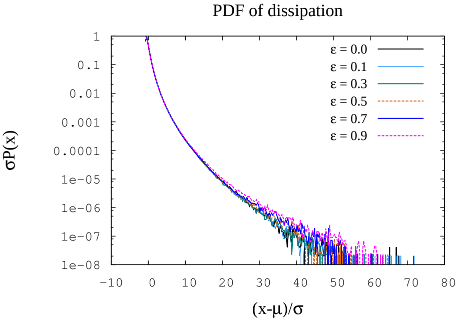


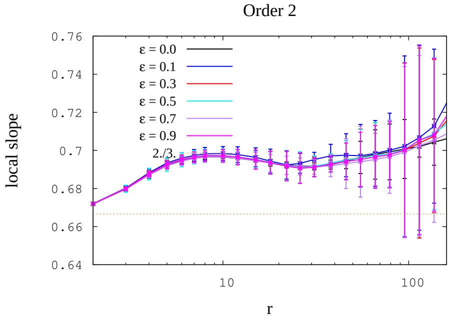


What happens if we allow the killed degrees of freedom to be the part of the dynamics?

Static decimation

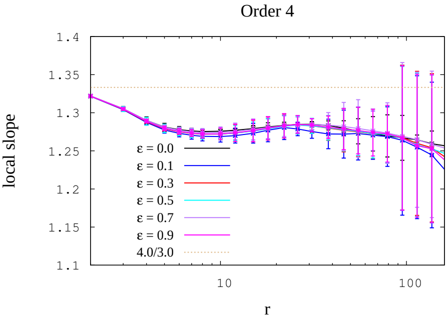
- Solve full Navier-Stokes equations and then apply projection on the fields to remove desired number of helical modes.

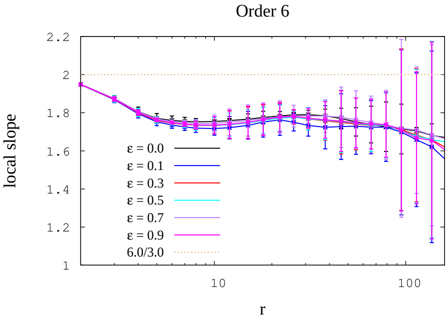




Results agree with full Navier-Stokes equations.

Static decimation





- There is no effect of static decimation of helical modes of statistics of Navier-Stokes equations.
- Only decimated helical modes not taking part in the dynamics change the statistics.

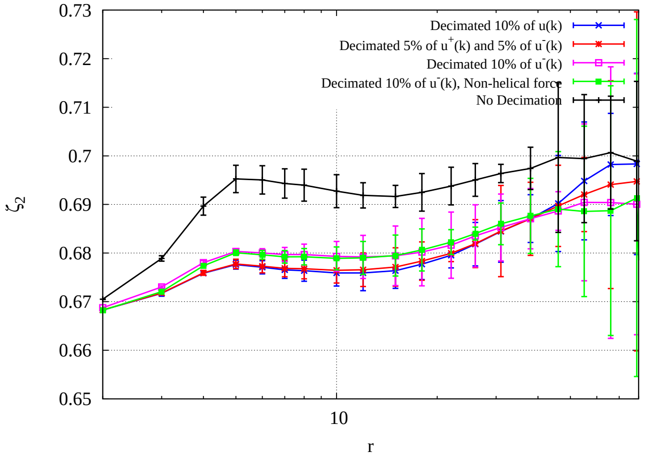
- In our simulations, transition from forward cascade to inverse cascade of energy occurs between $\epsilon = 0.99$ to $\epsilon = 0.999$.
- The critical value of ϵ close to 1 indicates that presence of only a small fraction of helical modes of other sign could reverse the dynamics.
- Intermittency reduces as we increase the fraction of the modes decimated of one helicity sign.
- Statistics differs if the decimated modes were the part of the dynamic evolution.

Thank you!

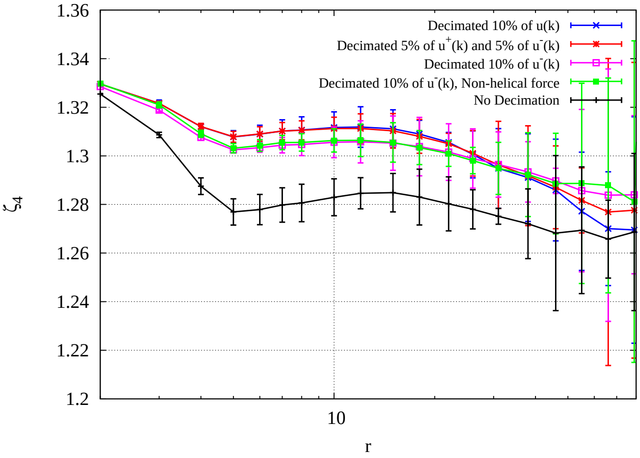
Support

- ERC Advanced Grant 'NewTurb', PI: Prof. Luca Biferale.

Multi scaling exponent ζ_2, local slope of $S_2(r)/S_3(r)$



Multi scaling exponent ζ_4, local slope of $S_4(r)/S_3(r)$



Multi scaling exponent ζ_6, local slope of $S_6(r)/S_3(r)$